

# *EXPLORING THE POTENTIAL OF “ALTERNATIVE GRAPHS” TO RESCHEDULE TRAINS*

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**[http://www.dia.uniroma3.it/~  
dariano/Brief\\_CV.pdf](http://www.dia.uniroma3.it/~dariano/Brief_CV.pdf)**

## Who is the speaker?

- **Andrea D'Ariano** is Associate Professor  
in *Operations Research (OR)*
- Background of knowledge in OR, Computer Science,  
Railway Engineering, Intelligent Transportation Systems
- Winner of Prizes by IEEE, INFORMS, IAROR, AIRO, IEOM, ...
- Associate Editor for well-known journals (Transp. Res. B, C, E)
- Participation in several research projects with Universities,  
Research Institutes, Transportation Companies and Organizations
- Coordinator of AIRO (Italian Assoc. of Operat. Research) Chapter  
on “*Optimization in Public Transport and Shared Mobility*”



# Railway Optimization: *Our group*

## Railway Operations Research @ Roma Tre

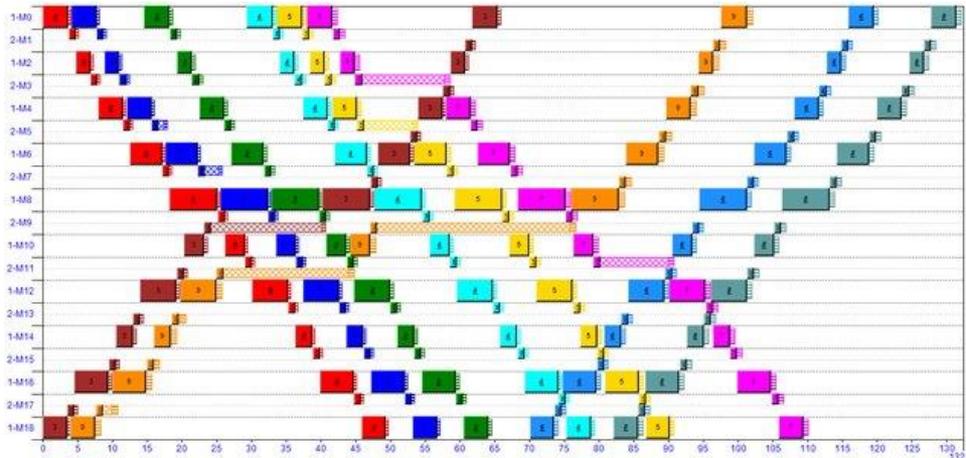


# What represents an Alternative Graph (AG)?

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  - Each *job* corresponds to a vehicle or person taking some actions
  - Each job is composed by a set of *operations* to be performed
  - The set of operations of each job can be pre-defined or flexible
  - Each operation is related to a job and a capacitated *resource*
  - Each resource is shared by different jobs in the schedule



[[Shi Qiang Liu](#),  
[Erhan Kozan](#),  
Transp. Science  
2011]

# What represents an Alternative Graph (AG)?

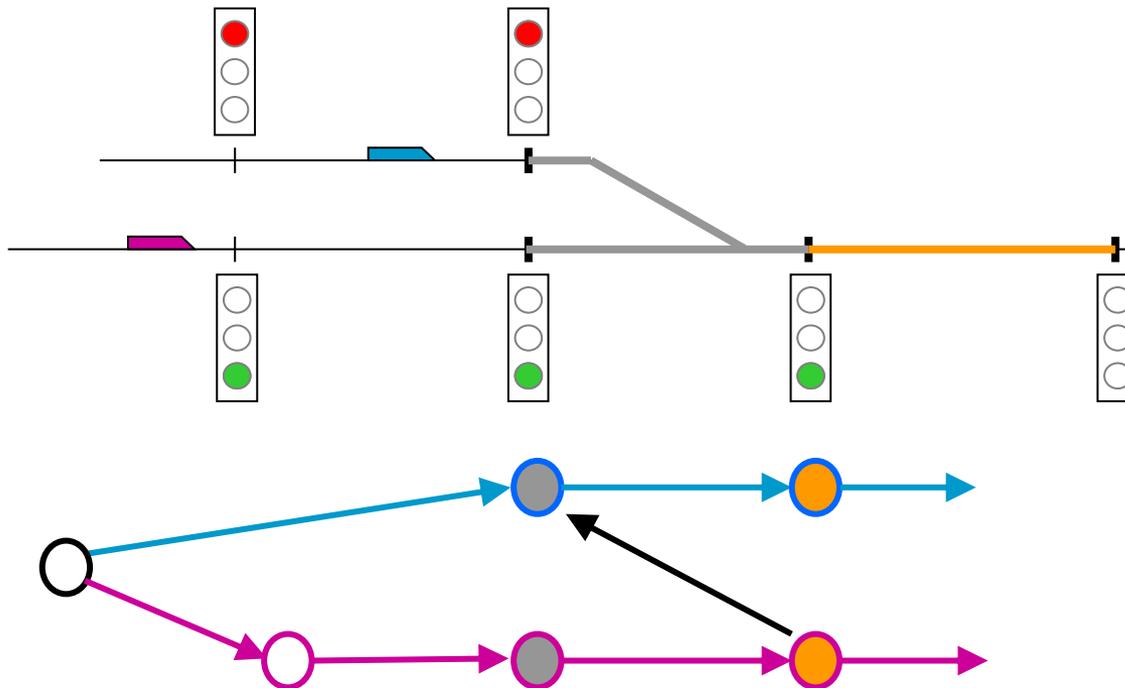
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  - Each operation is related to a job and a capacitated *resource*
  - Each resource is shared by different jobs in the schedule
- JSSP can easily represent a **train scheduling problem** in which:
  - Each job corresponds to a specific *train*
  - Each resource corresponds to a piece of *railway track*
  - Each *operation* is a piece of track that is occupied by a train
  - The set of operations of a job is the *train routing*

# What represents an Alternative Graph (AG)?

- **AG** is particularly suitable to model train scheduling problems.  
In AG, each *node* is an operation, while each *arc* is a constraint

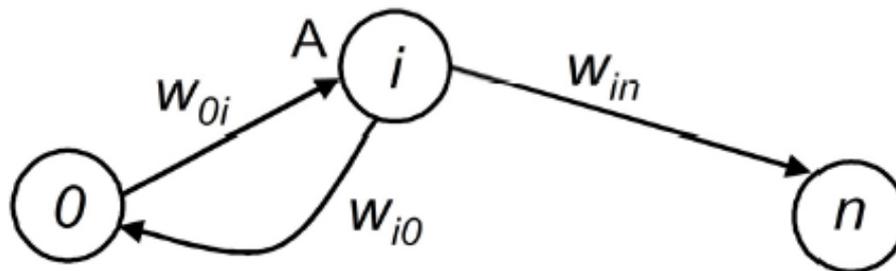
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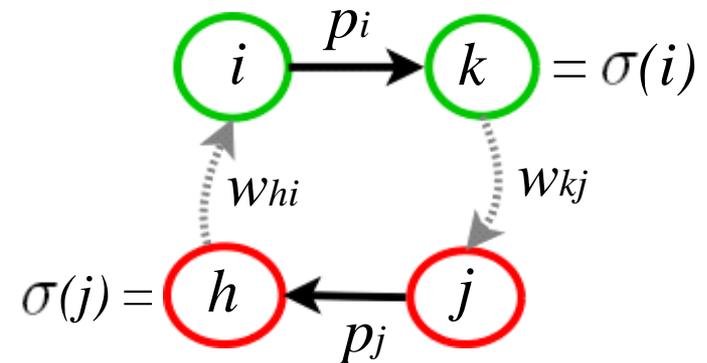
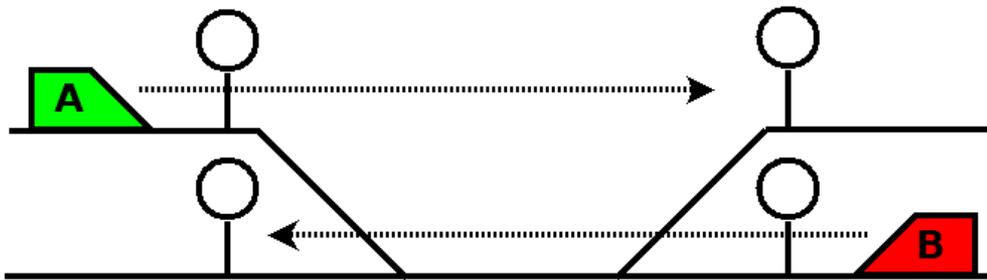
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- Each train has a travel time window according to the timetable, i.e. minimum & maximum times to start processing an operation, requiring *release & due date* (soft) or *deadline* (hard) constraints
- **Other types of constraints:** *service connection* constraints, *rolling stock* constraints, *arrival and departure time* constraints, *resource availability* constraints, *min and max travel time* constraints, ...

# What about the modeling assumptions?

- Each operation has a start time (i.e. a **timing variable**) and a duration time (input data), requiring a pre-defined processing time

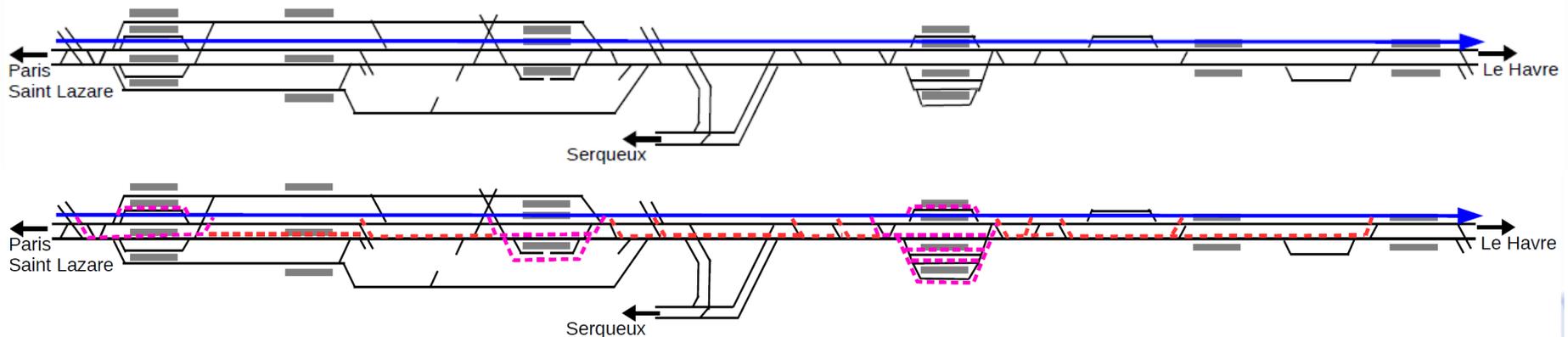
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- Each operation has a start time (i.e. a timing variable) and a duration time (input data), requiring a pre-defined processing time
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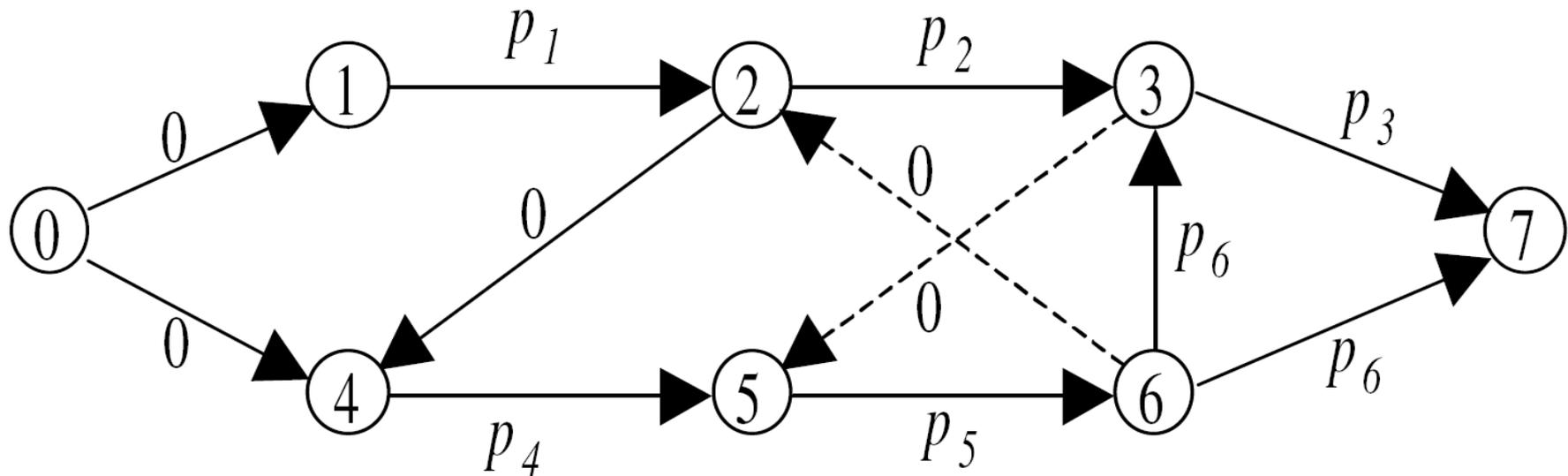
[Source: Paola Pellegrini]

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- ❑ The routing of each train can be either fixed or flexible (each job can be a variable), with possibility of local or global re-routing
- ❑ The train arrival and departure times can also be flexible
- ❑ Travel/dwell times are constrained between min and max values
- ❑ Assumptions on time and resource granularities must be set

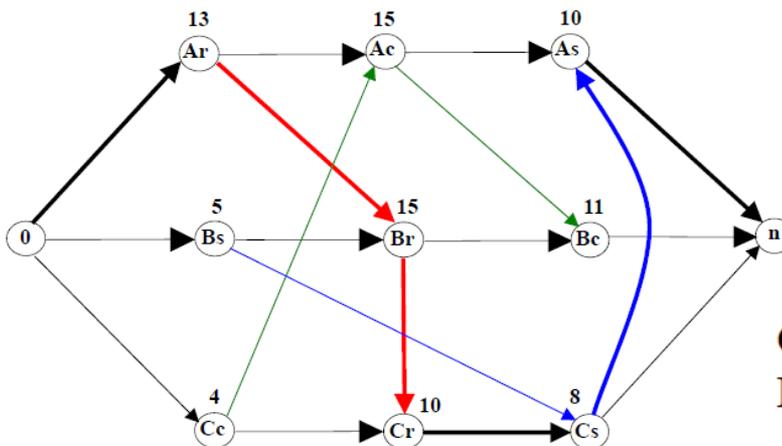
## What about the modeling assumptions?

- The problem complexity (finding a **feasible schedule** is *NP-hard*) depends on the assumptions regarding the granularity, i.e. on the number of sequencing and routing variables (the timing variables are easy to handle, since modelled as shortest path problems).



## What about the modeling assumptions?

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- The objective function is usually related to the timing of operations. There are powerful scheduling-theory-based techniques to minimize the maximum completion time or delay.



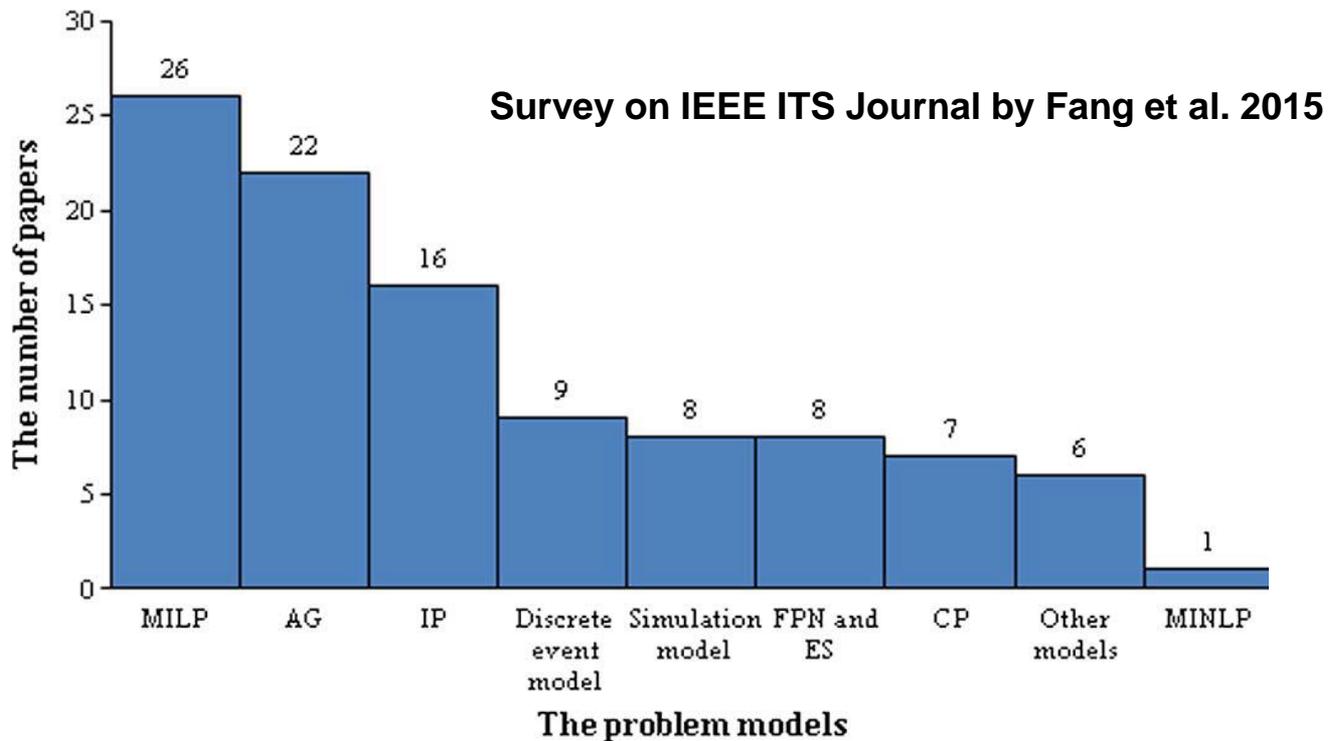
**One critical path: Ar – Br – Cr – Cs – As**  
**Makespan: 13+15+10+8+10 = 56**

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- The objective function is usually related to the timing of operations. There are powerful scheduling-theory-based techniques to minimize the maximum completion time or delay.
- Other objective functions are possible, but the resulting problems might be more difficult to handle with AG, while more general mathematical formulations can easily incorporate them (even if general solvers might be slow to converge to near-optimum).

# Which models exist in the literature?

- A significant number of papers use AG for train scheduling:



- Two main streams of research are based on either resource-dependent (e.g., MILP) or time-dependent formulations. Their complexity depends on the adopted resource and time granularity.

# Which solving methods exist?



## □ General (commercial) solver:

- Pros: easy to formulate business rules and objectives
- Contros: very slow solving process when increasing problem size

## □ Smart (problem-dedicated) solver:

- Pros: very good performance and scalability
- Contros: some business rules and objectives require a lot of work

AG-based software uses heuristic, meta-heuristic, and exact algorithms to handle different types of variables. These algorithms need to be adapted when changing constraints/rules and objectives.

Pre-processing is a key factor for any solver, e.g. filtering the train routes, pre-selecting variable values, reducing the variables.

# Which types of problem decomposition?

- Decomposition is needed in practice and can be of different types:
  - Temporal decomposition, e.g., rolling horizon or MPC approaches;
  - Spatial decomposition, e.g., coordination or Benders approaches;
  - Decomposition based on the different types of variables, e.g., timing, sequencing, and routing approaches;
  - Decomposition based on different decision layers, e.g., the variables are grouped based on the definition of sub-problems.

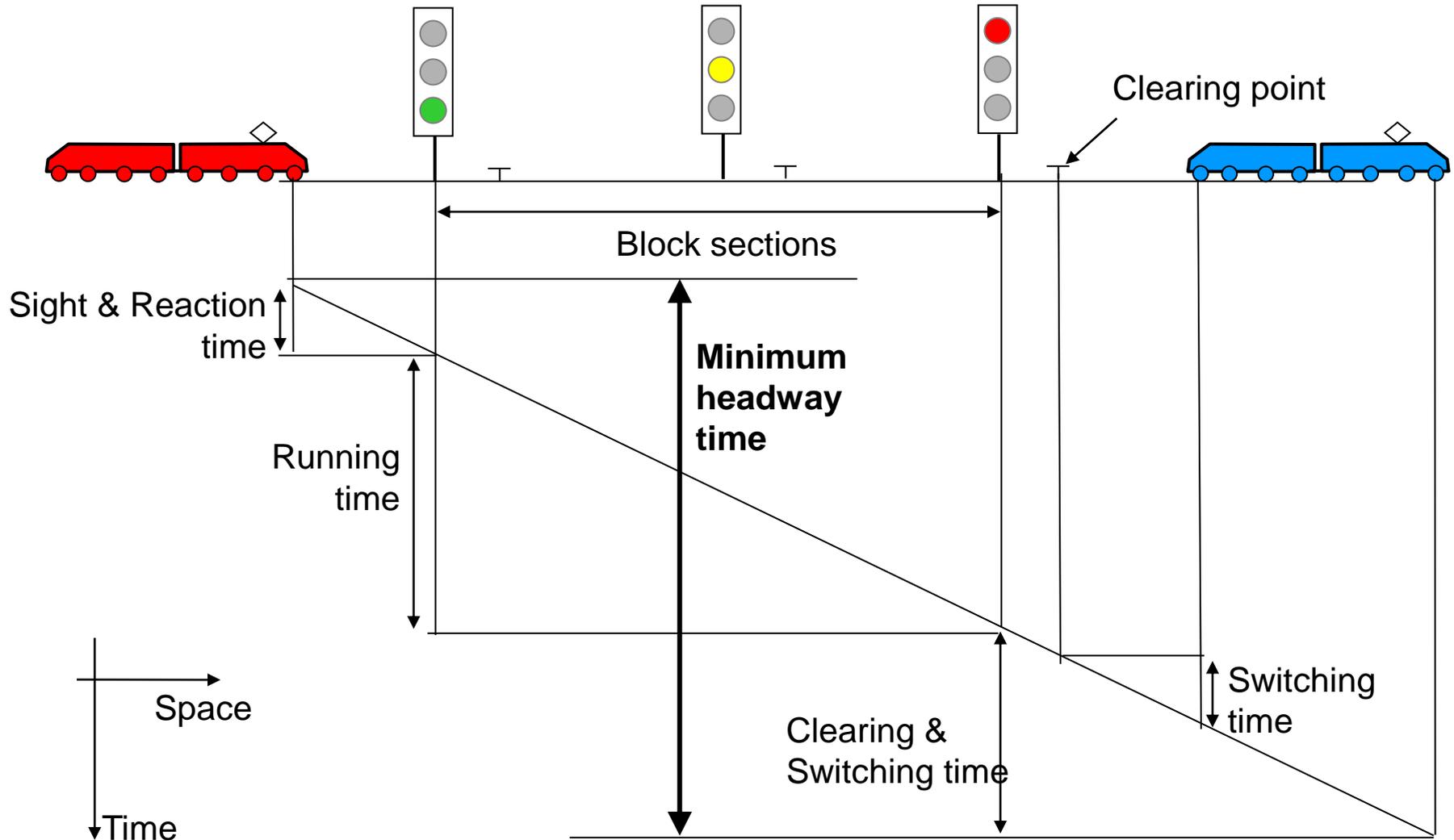
All the decomposition methods are iterative and require to study *convergence*, *performance*, and *scalability* factors.



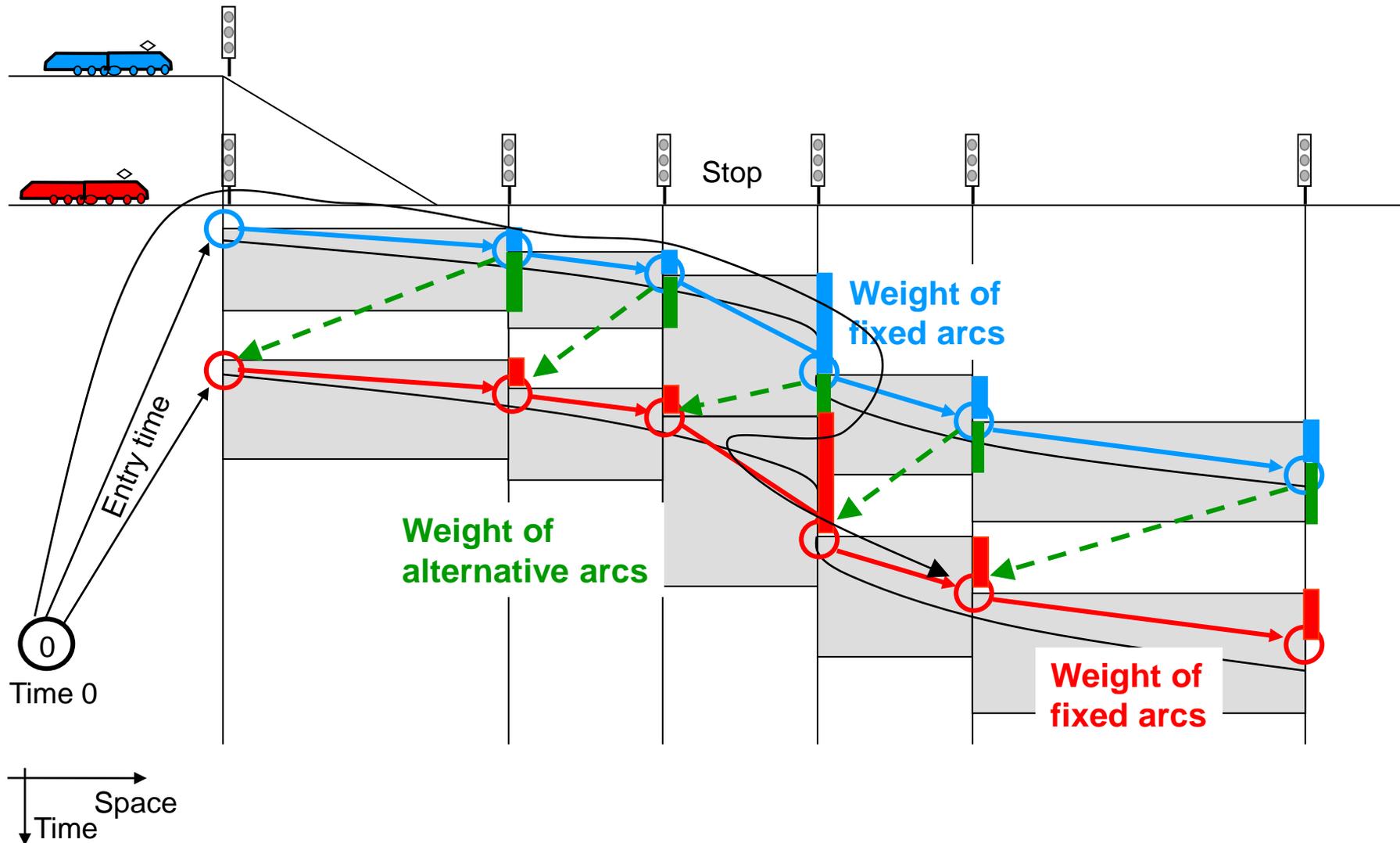
# Train Rescheduling Problem

- Aim:** Development of novel railway traffic management systems for a timely, precise and effective train traffic regulation in terms of punctuality increase and energy efficiency
- Tool:** Flexible rail operations via advanced models and algorithms for optimizing train sequencing, routing and timing decisions
- Application:** Recover real-time railway traffic disturbances such as multiple delayed trains and blocked tracks

# Background: Blocking time theory



# Conflict Detection and Resolution (CDR)

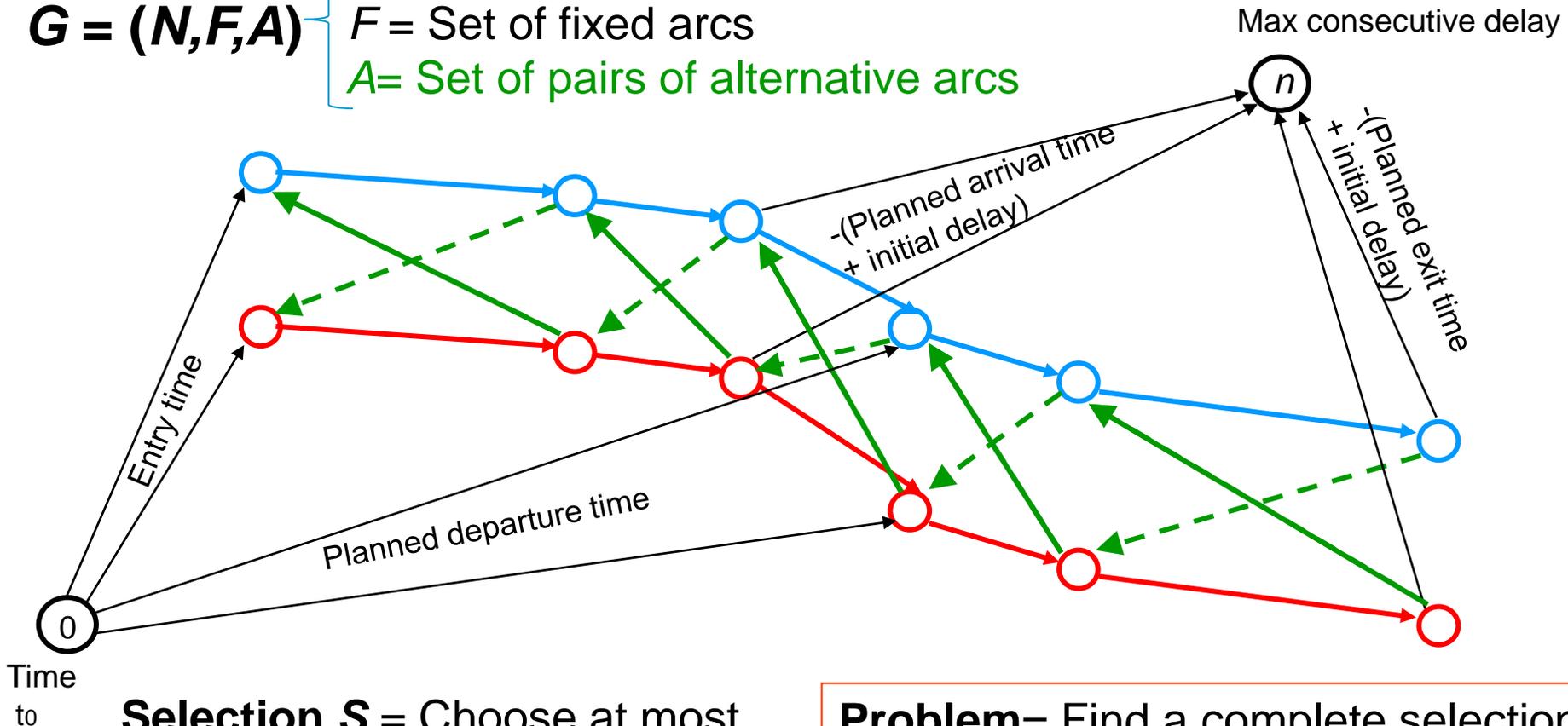


# Alternative Graph (AG)

[Mascis  
Pacciarelli  
EJOR 2002]

$G = (N, F, A)$

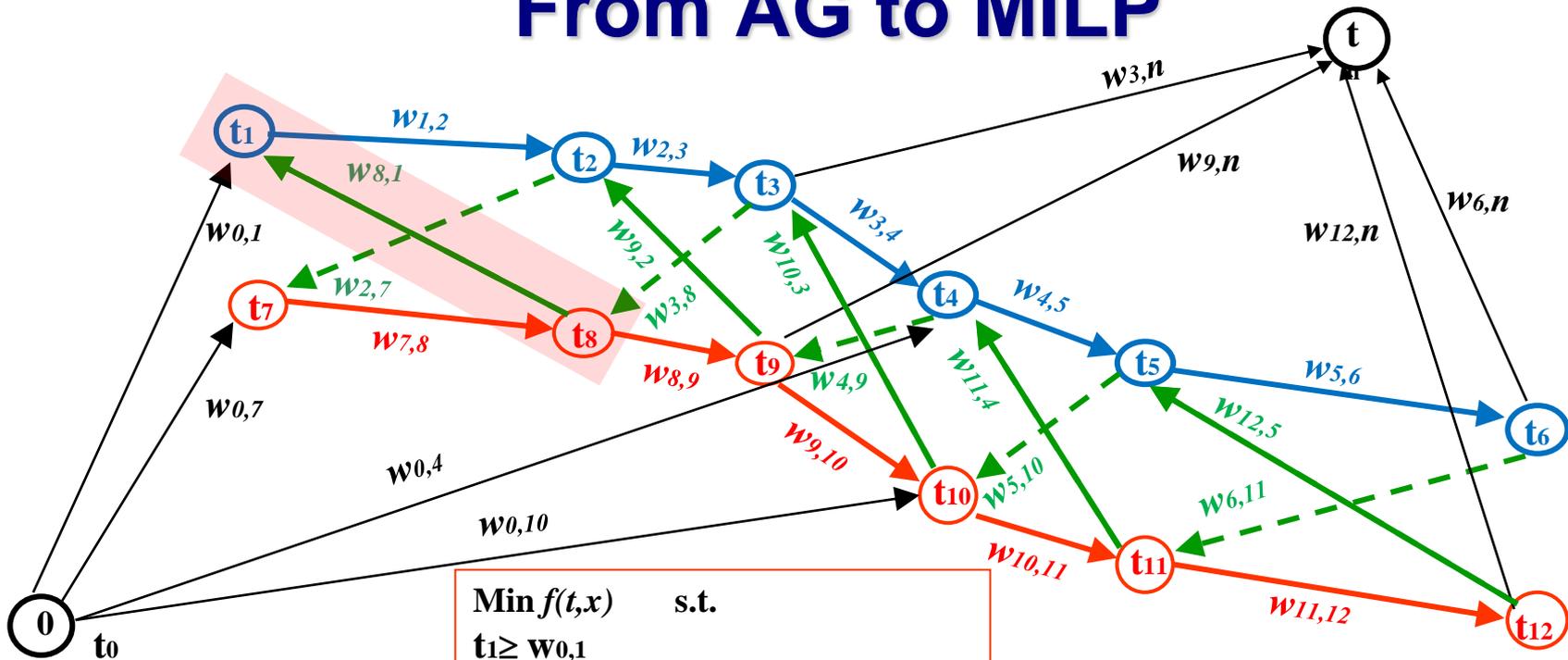
- $N$  = Set of nodes
- $F$  = Set of fixed arcs
- $A$  = Set of pairs of alternative arcs



**Selection S** = Choose at most one arc from each pair in  $A$ , thus obtaining a graph  $G(S) = (N, F \cup S)$

**Problem** = Find a complete selection  $S$  such that the longest path from 0 to  $n$  in  $G(S)$  is minimum

# From AG to MILP

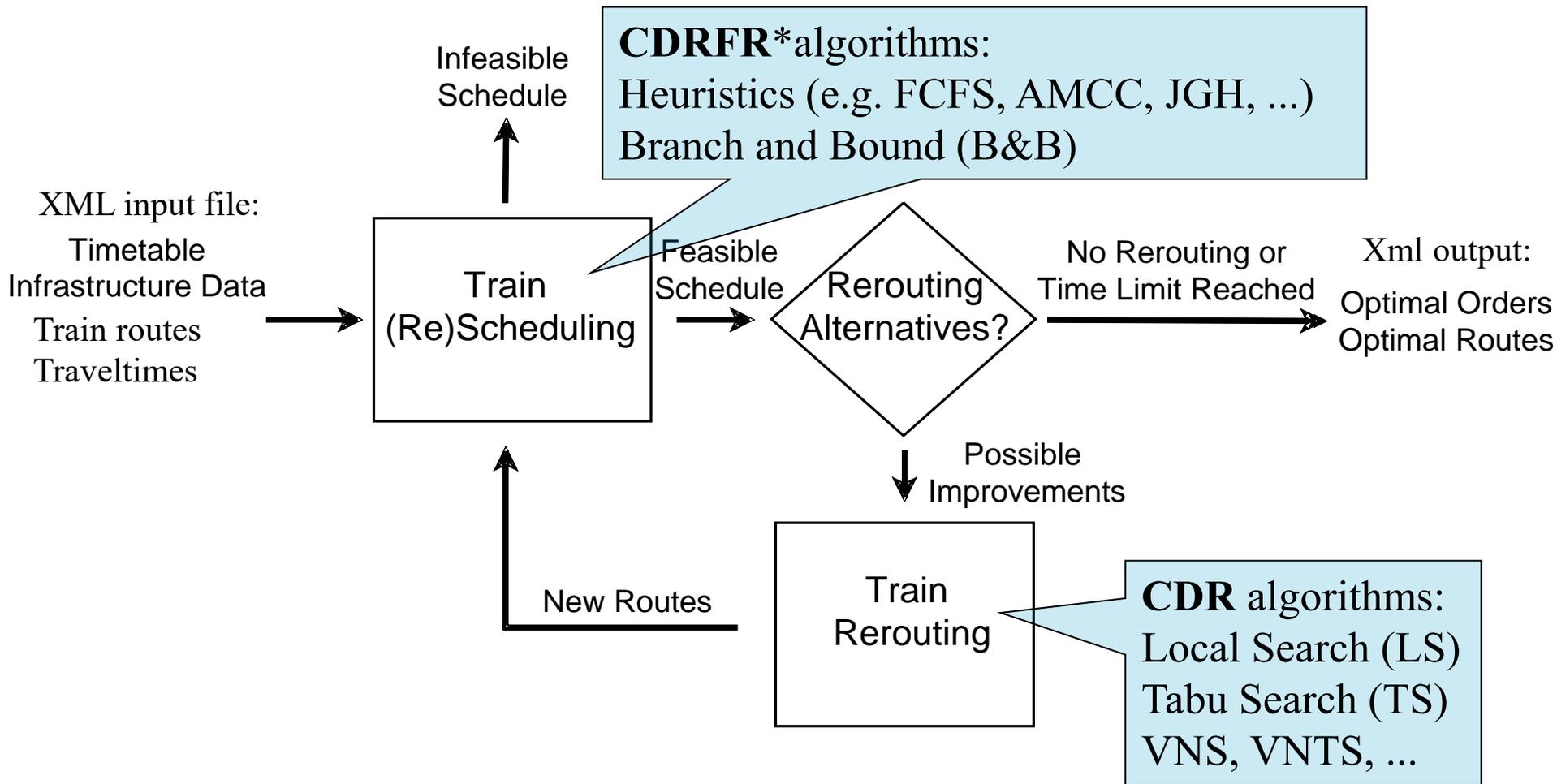


Min  $f(t,x)$  s.t.

$t_1 \geq w_{0,1}$   
 $t_7 \geq w_{0,7}$   
 $t_4 \geq w_{0,4}$   
 $t_{10} \geq w_{0,10}$   
 $t_2 \geq t_1 + w_{1,2}$   
 ...  
 $t_{12} \geq t_{11} + w_{11,12}$   
 $t_1 \geq t_8 + w_{8,1} - M(1 - X_{8,1,2,7})$   
 $t_7 \geq t_2 + w_{2,7} - MX_{8,1,2,7}$   
 ...

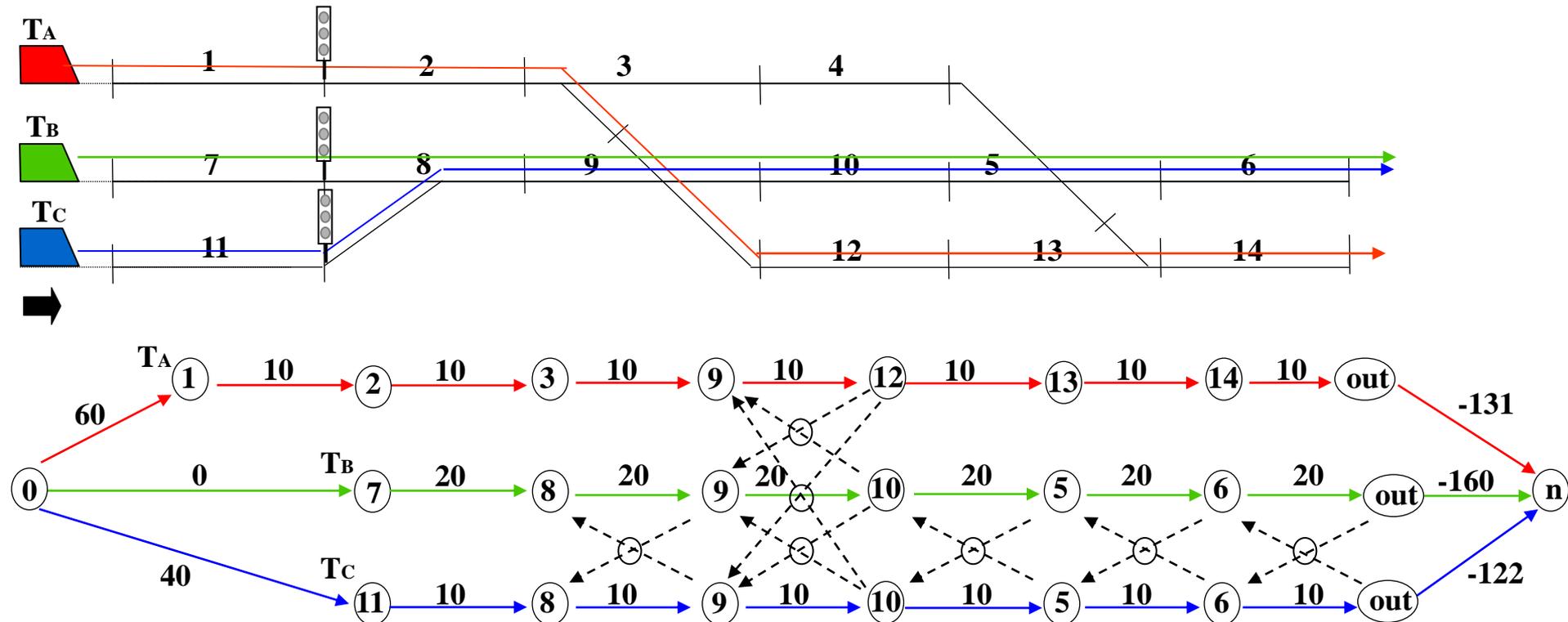
$X_{8,1,2,7} = 1$   
 $X_{9,2,3,8} = 1$   
 $X_{10,3,4,9} = 1$   
 $X_{11,4,5,10} = 1$   
 $X_{12,5,6,11} = 1$

# Optimization software: AGLIBRARY



# Illustrative example (1)

CDRFR formulation of a small example with three trains



Each alternative pair is used to order two trains on a block section

# Branch and bound algorithm

[D'Ariano  
EJOR 2007]

**Branching rule:** Choose the most critical unselected alternative pair and branch on this pair.

**Hybrid search strategy:** Alternate  $X$  repetitions of the depth-first visit with the choice of the open node of the search tree with smallest lower bound among the last  $Y$  open nodes.

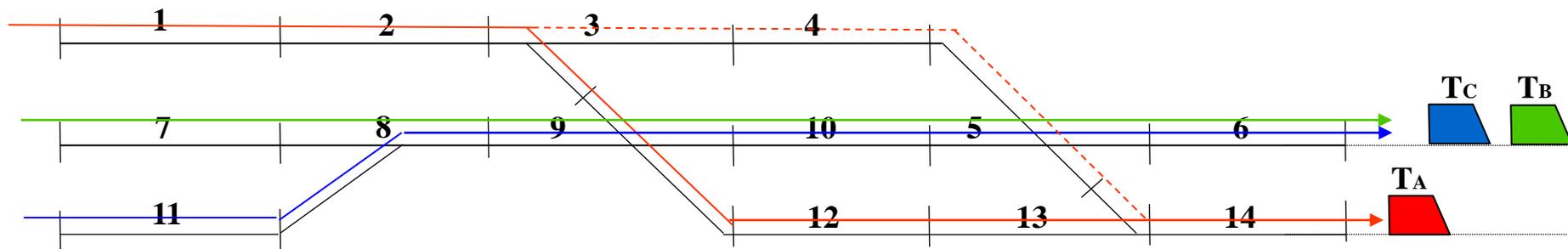
**Lower bound method:** Generalization of the “Jackson pre-emptive schedule” [Carlier & Pinson MS 1989]. Implementation + evaluation of single and parallel machines [Brucker & Brinkkötter JS 2001].

**Implications rules:** Network topology (static/off-line rules) and alternative graph proprieties (dynamic/on-line rules)

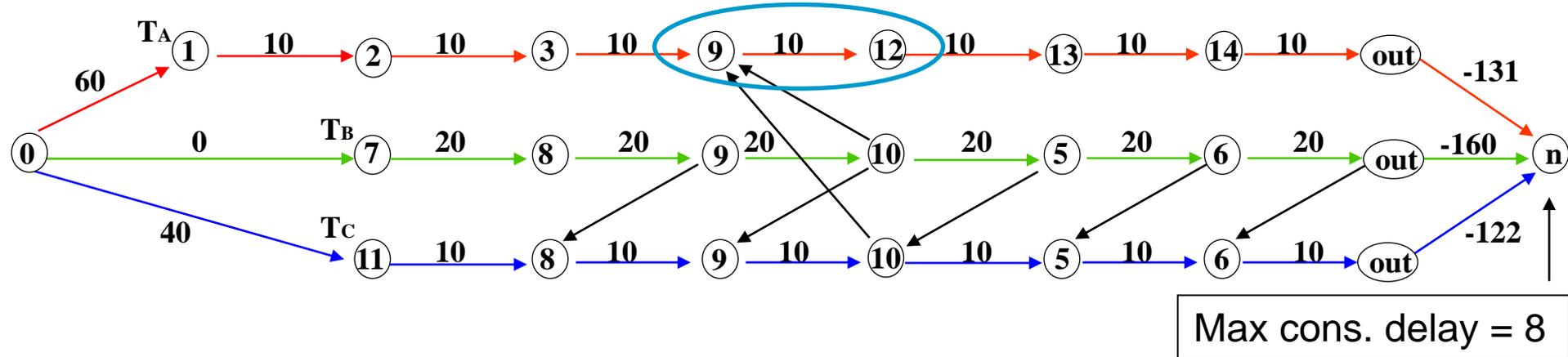
# Illustrative example (2)

[D'Ariano  
EJOR 2007]

Optimal CDRFR solution computed by the B&B algorithm



local rerouting available...

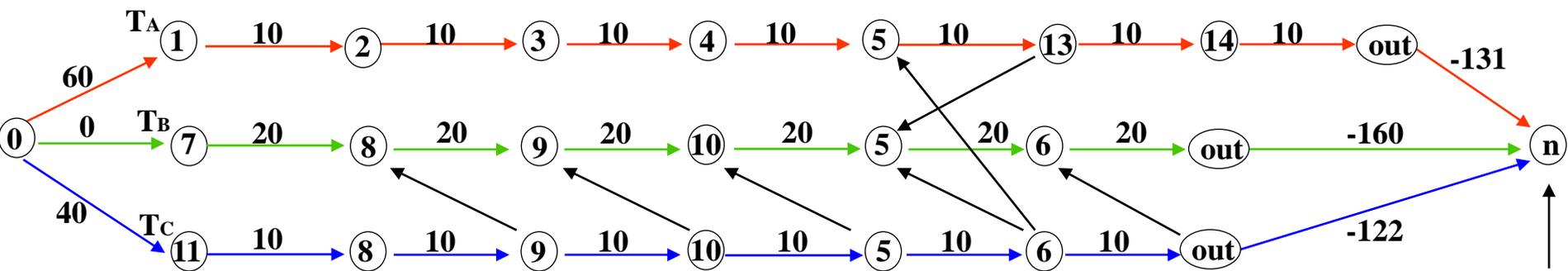
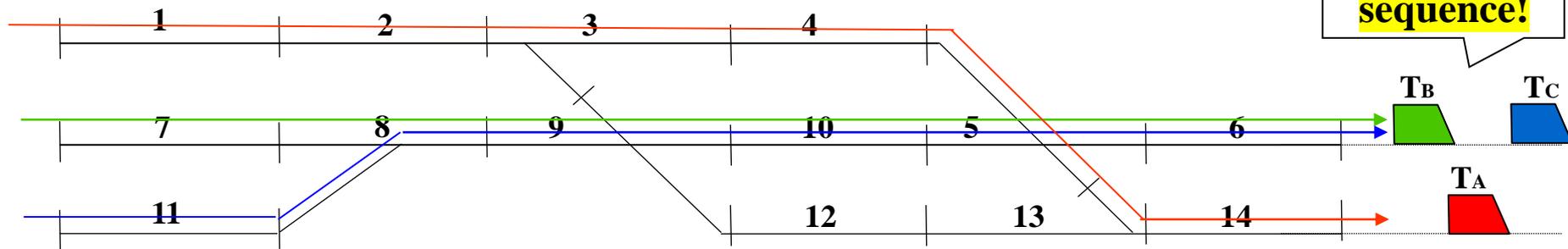


A conflict-free deadlock-free schedule is a complete consistent selection  $S$

# Illustrative example (3)

Optimal solution to the compound CDR problem

New output sequence!



Max cons. delay = 0

A new route for TA and a new complete consistent selection S are shown

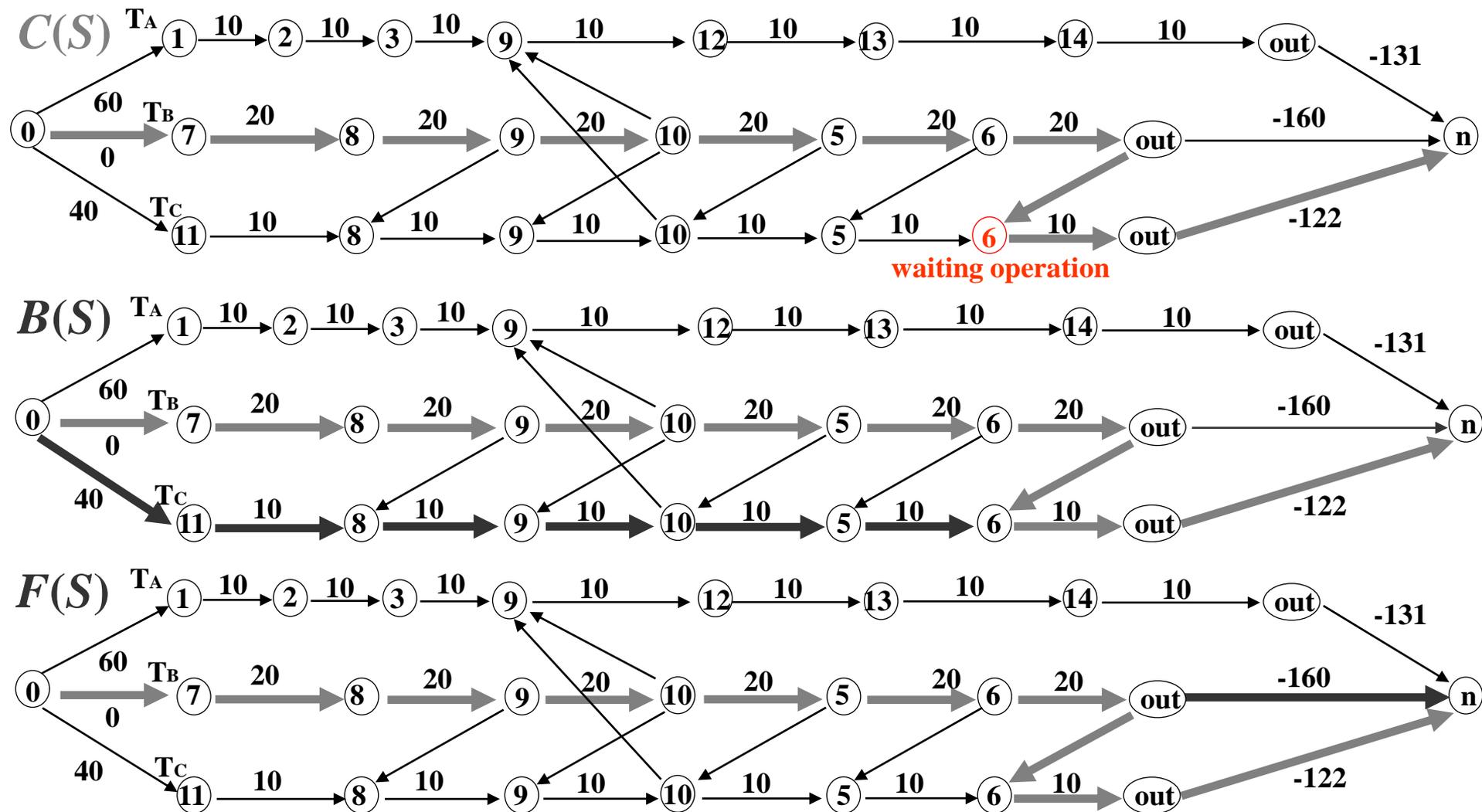
# CDR: Move & neighbourhood

[D'Ariano  
Transport.  
Science 2008]

We start from the solution obtained for the CDR problem with fixed routes. A **local search** for better train routes is as follows:

- A **move** is to change one route and its **evaluation** is to solve the associated CDRFR problem;
- At each iteration the best (local) move is taken from a set of **neighbours** of a current CDR solution;
- **Neighbourhood**: It is well known that a solution can be improved by *changing* the critical path related to the current selection  $S$  only;
- Our local search is based on a ramified critical paths in order to select potentially improving routes.

# Illustrative example of ramified critical paths

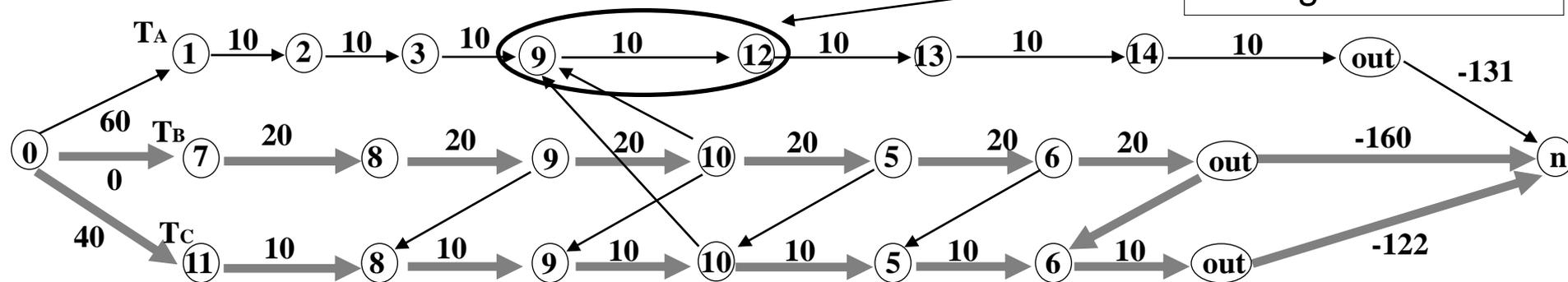


# CDR: Tabu search algorithm

[Corman  
TRpB 2010]

The ramified critical paths are well focused on reducing the maximum consecutive delay but are not always opt-connected.

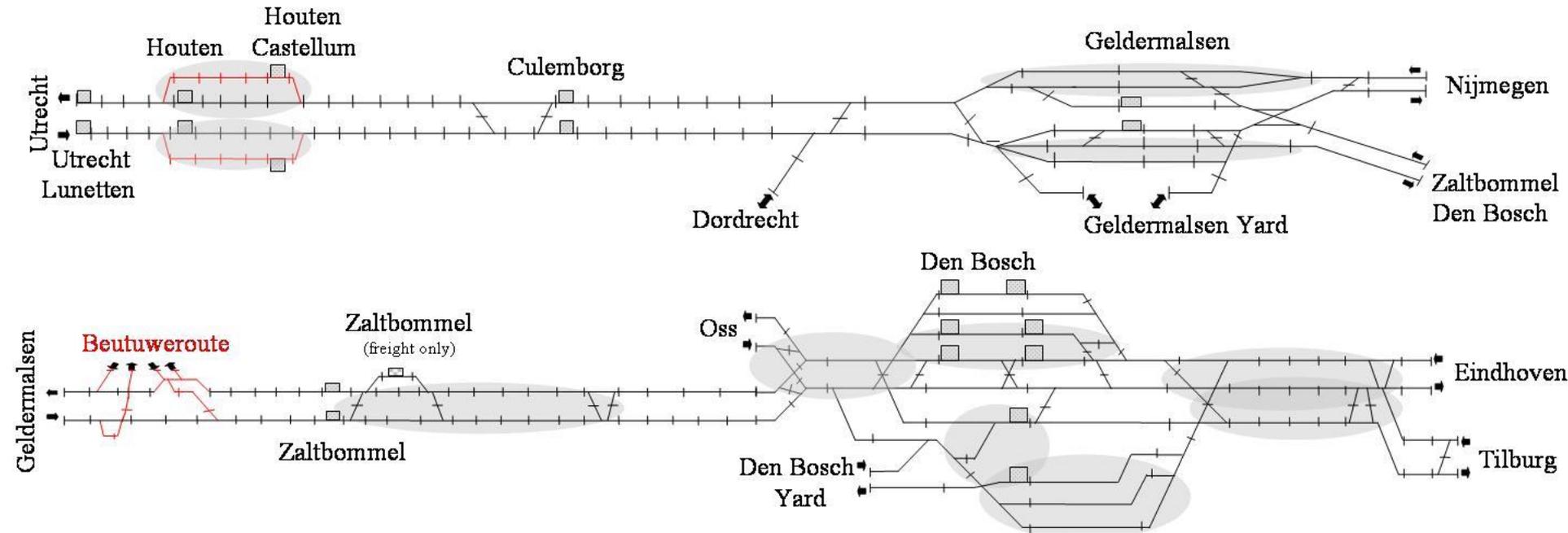
The best rerouting belongs to TA!



A novel **tabu search** (TS) algorithm escapes from local minima by taking a non-improving move and then forbidding the inverse move for a given number of iterations.

Another technique to escape from local minima is based on **restarts** (i.e., performing a few moves regardless they are good or bad).

# Test on a Dutch train dispatching area

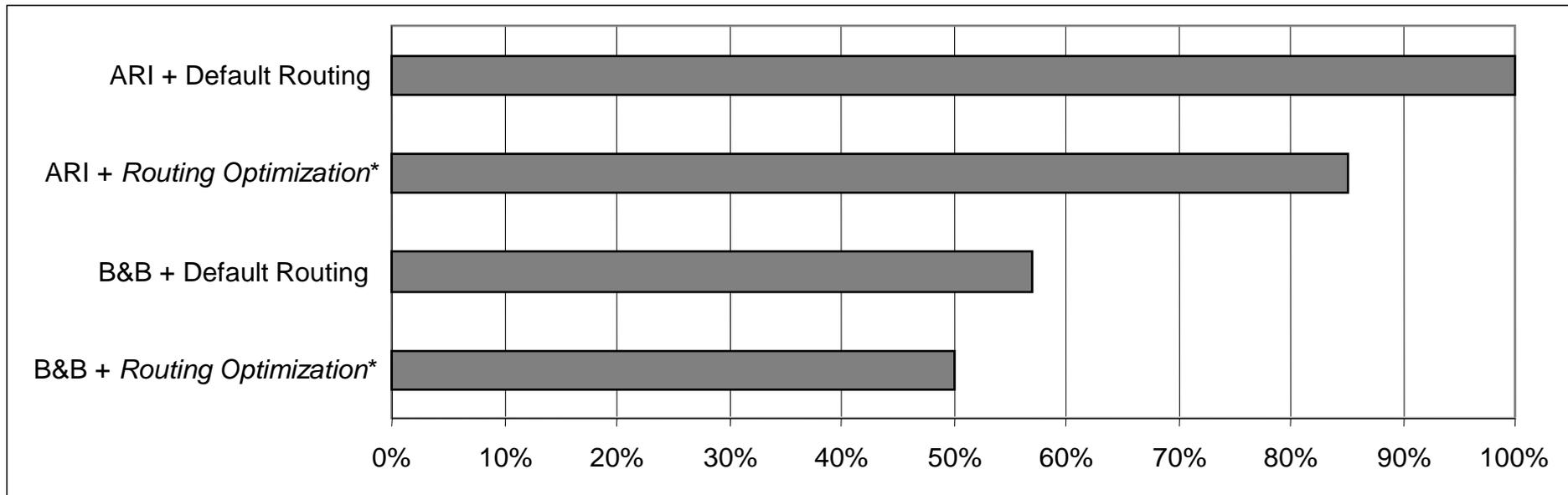


- ❑ Utrecht-Den Bosch railway network (50 km long, including 21 station platforms)
- ❑ 40 running trains per hour (timetable 2007)
- ❑ Rolling stock connections are located in Zaltbommel and Den Bosch stations
- ❑ Rerouting is performed in stations and corridors (356 local routes)

# Results on the compound CDR problem (1)

[D'Ariano Transp. Science 2008]

Percentage of maximum consecutive delays for four ROMA(AGLIBRARY) config.



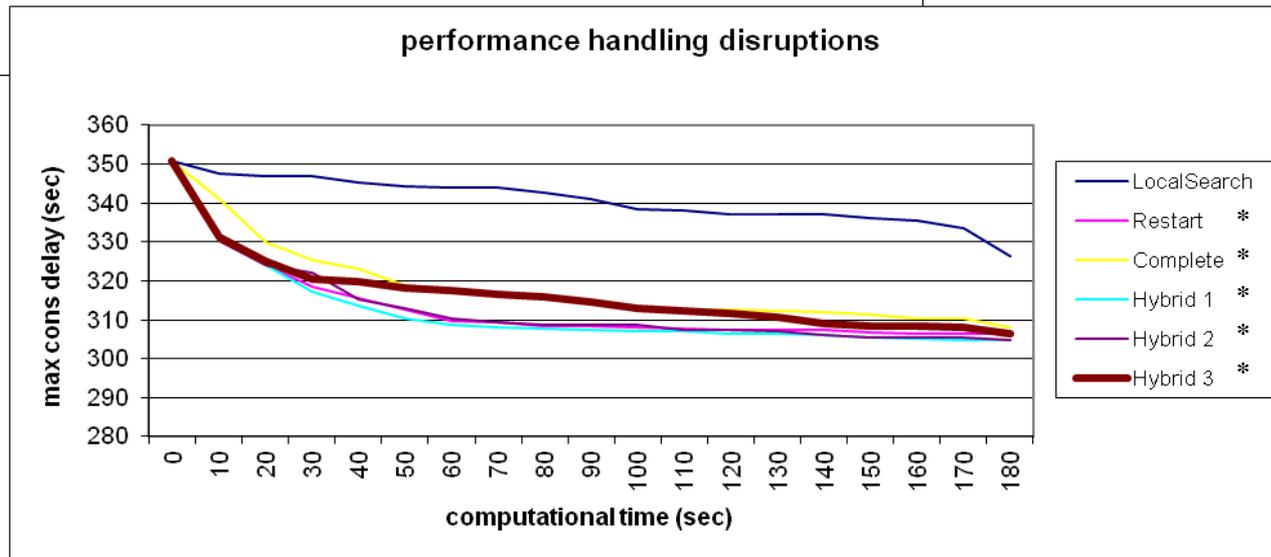
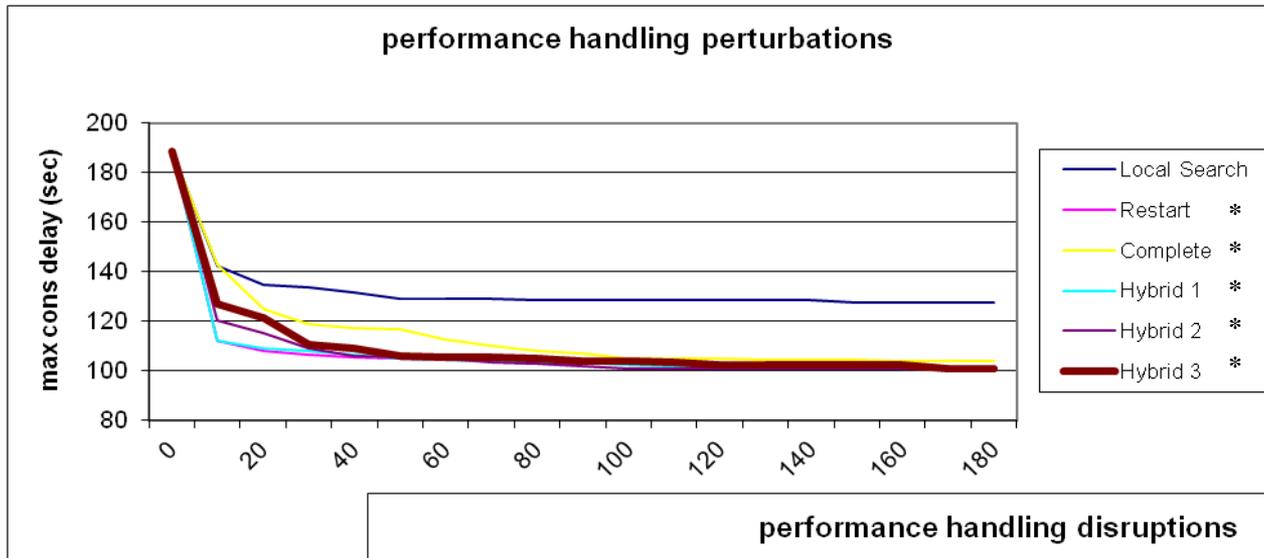
Average Results (in seconds)	Default		Routing	Routing Optimization*		
	Delay Max	Delay Avg	Time Tot	Delay Max	Delay Avg	Time Tot
ARI	489.4	66.9	0.6	417.0	60.5	8.1
B&B	279.8	50.4	2.1	245.3	44.8	33.9

\*Routing Optimization by the local search algorithm

# Results on the compound CDR problem (2)

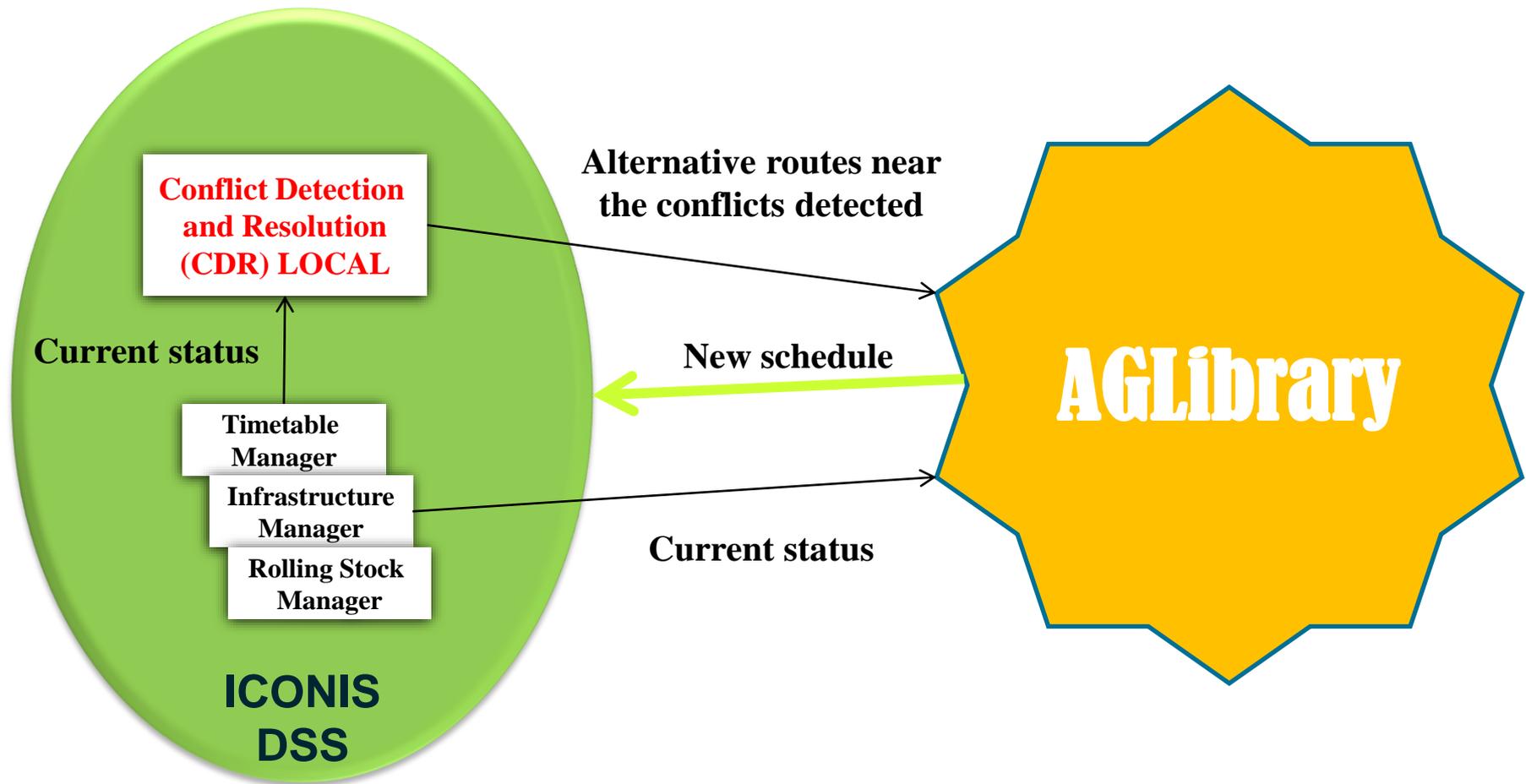
[Corman  
TRpB 2010]

**Perturbations  
are multiple  
train delays**



**Disruptions  
are tracks  
which are  
blocked**

# Alstom Strategy



# Railway network (nearby London)





# Computational results

**Intel Core 2 Duo E6550 (2.33 GHz), 2 GB di RAM, Windows XP**

**Scheduling & routing problem (CDR problem) : 29 instances**

**CPLEX (algorithm: 1 hour of computation):**

**[MILP formulation solved by IBM LOG CPLEX MIP 12.0]**

**❖ 6 fails, 22 optimum, avg comp time (algo) best sol 1011.7 sec**

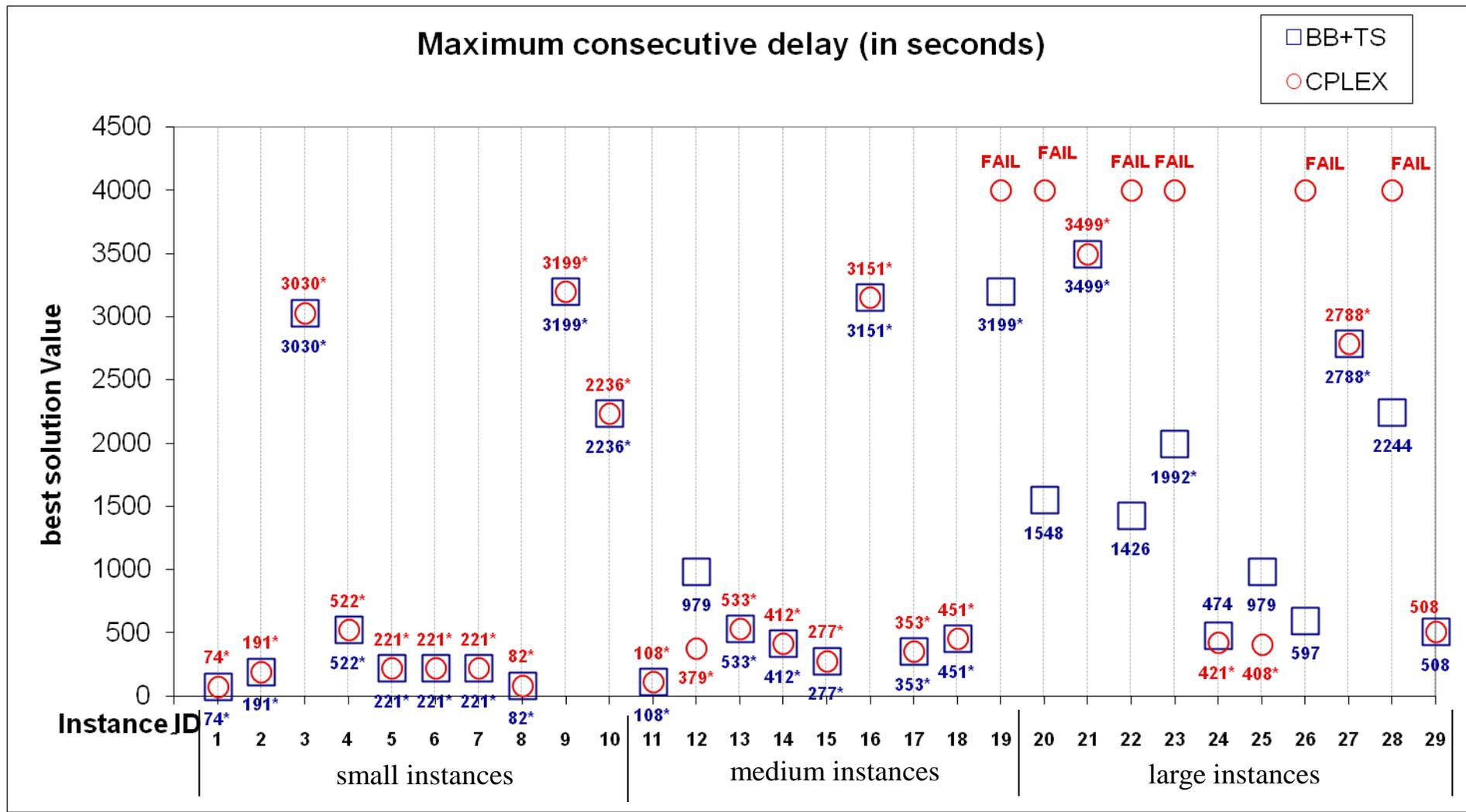
**AGLIBRARY\* (algorithm: 20 sec of computation):**

**[Branch & Bound (EJOR, 2007) + Tabu Search (TRpartB, 2010)]**

**❖ 0 fails, 21 optimum, avg comp time (algo) best sol 11.6 sec**

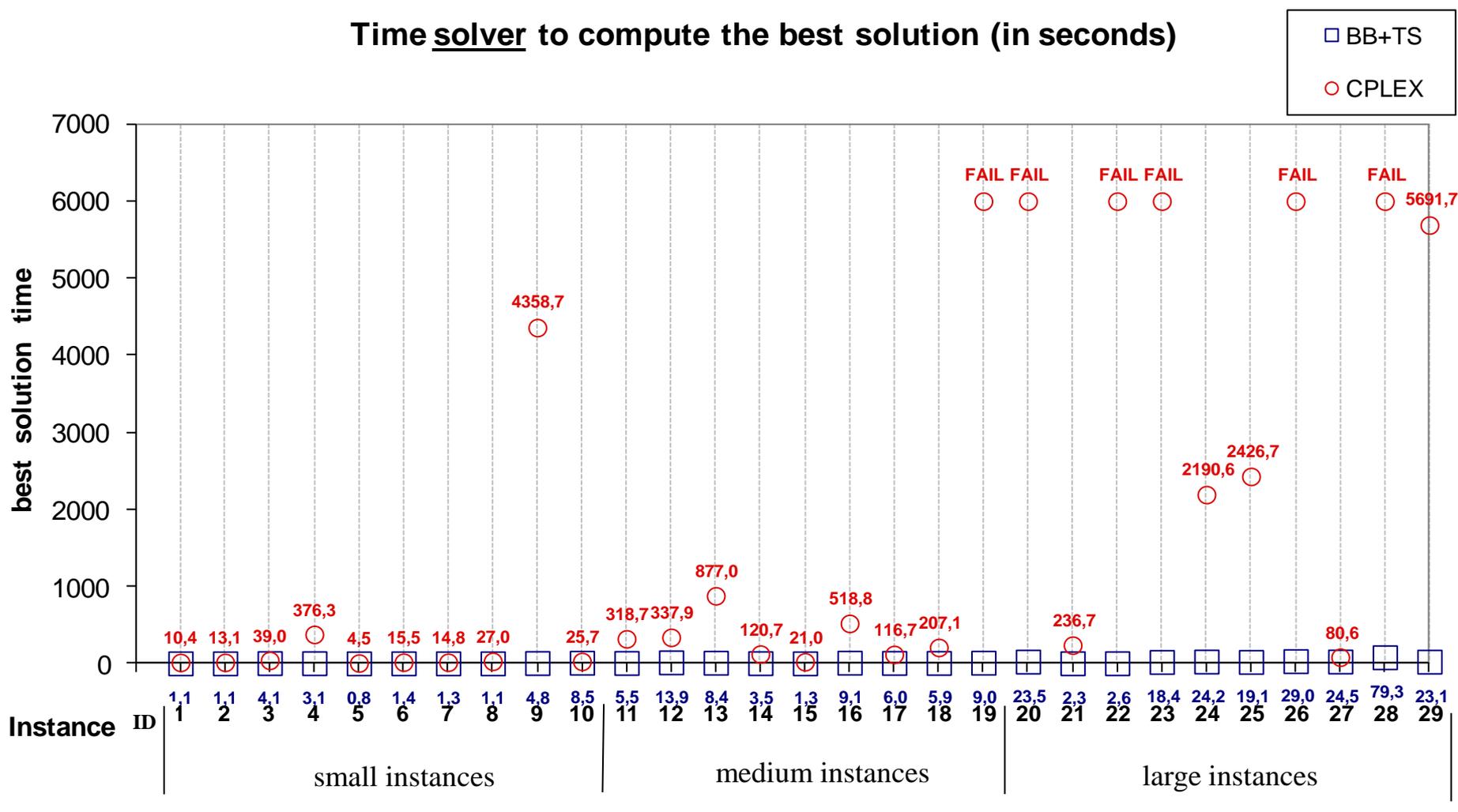
***\*Better computat. results are obtained with VNS (Samà COR 2017)***

# CPLEX vs AGLIBRARY (scheduling & routing)



# CPLEX vs AGLIBRARY (scheduling & routing)

Time solver to compute the best solution (in seconds)



## A list of recent publications on railway operations research:

- M. Samà, P. Pellegrini, A. D'Ariano, J. Rodriguez, D. Pacciarelli (2017) On the tactical and operational train routing selection problem, *Transportation Research, Part C*, 76(1) 1–15,
- F. Corman, A. D'Ariano, A.D. Marra, D. Pacciarelli, M. Samà (2017) Integrating Train Scheduling and Delay Management in Real-time Railway Traffic Control, *Transportation Research, Part E*, 105, 213–239
- M. Samà, A. D'Ariano, F. Corman, D. Pacciarelli (2017) A variable neighborhood search for fast train scheduling and routing during disturbed railway traffic situations, *Computers and Operations Research*, 78(1) 480–499
- M. Samà, P. Pellegrini, A. D'Ariano, J. Rodriguez, D. Pacciarelli (2016) Ant colony optimization for the real-time train routing selection problem, *Transportation Research, Part B*, 85(1) 89–108
- M. Samà, C. Meloni, A. D'Ariano, F. Corman (2015) A multi-criteria decision support methodology for real-time train scheduling. *Journal of Rail Transport Planning & Management*, 5(3) 146–162
- A. D'Ariano, M. Samà, P. D'Ariano, D. Pacciarelli (2014) Evaluating the applicability of advanced techniques for practical real-time train scheduling. *Transportation Research Procedia*, 3 279–288,
- G.L. Giacco, D. Carillo, A. D'Ariano, D. Pacciarelli, A.G. Marin (2014) Short-term rail rolling stock rostering and maintenance scheduling. *Transportation Research Procedia*, 3 651–659,
- T. Dollevoet, F. Corman, A. D'Ariano, D. Huisman (2014) An iterative optimization framework for delay management and train scheduling. *Flexible Services and Manufacturing Journal*, 26(4) 490–515,
- F. Corman, A. D'Ariano, D. Pacciarelli and M. Pranzo (2014) Dispatching and coordination in multi-area railway traffic management. *Computers and Operations Research*, 44(1) 146–160
- R. Larsen, M. Pranzo, A. D'Ariano, F. Corman, D. Pacciarelli (2014) Susceptibility of Optimal Train Schedules to Stochastic Disturbances of Process Times. *Flexible Services and Manufacturing Journal*, 26(4) 466–489
- G.L. Giacco, A. D'Ariano, D. Pacciarelli (2014) Rolling stock rostering optimization under maintenance constraints. *Journal of Intelligent Transport Systems: Technology, Planning, and Operations*, 18(1) 95–105
- F. Corman, A. D'Ariano, I.A. Hansen (2014) Evaluating disturbance robustness of railway schedules. *Journal of Intelligent Transport Systems: Technology, Planning, and Operations*, 18(1) 106–120
- G.L. Giacco, D. Carillo, A. D'Ariano, D. Pacciarelli (2014) Short-term rolling stock rostering and maintenance scheduling optimization. *Ingegneria Ferroviaria*, 1 39–52, Collegio Ingegneri Ferroviari Italiani.

- Corman, F., D'Ariano, A. (2013). Assessment of advanced dispatching measures for recovering disrupted railway traffic situations, *Transportation Research Record, Journal of the Transportation Research Board*, 2289, 1–9.
- R.M.P. Goverde, F. Corman, A. D'Ariano (2013) Railway line capacity consumption of different railway signalling systems under scheduled and disturbed conditions. *Journal of Rail Transport Planning & Management*, 3(3) 78–94
- Kecman, P., Corman, F., D'Ariano, A., Goverde, R.M.P. (2013). Rescheduling models for railway traffic management in large-scale networks. *Public Transport: Planning and Operations*, 5 (1–2) 95–123.
- Lamorgese, L., Mannino, C. (2015). An exact decomposition approach for the real-time train dispatching problem, *Operations Research* 63 (1), 48-64.
- Fang, W., Yang, S., Yao, X. (2015). A survey on problem models and solution approaches to rescheduling in railway networks, *IEEE Transactions on Intelligent Transportation Systems*, 16 (6) 2997–3016.
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- Liu, S.Q., Kozan, E. (2011) Scheduling trains with priorities: a no-wait blocking parallel-machine job-shop scheduling model, *Transportation Science* 45 (2), 175–198.
- Corman, F., D'Ariano, A., Pacciarelli, D., Pranzo (2010). M. A tabu search algorithm for rerouting trains during rail operations. *Transportation Research Part B*, 44 (1), 175–192.
- D'Ariano, A. (2010). Improving real-time train dispatching performance: Optimization models and algorithms for re-timing, re-ordering and local re-routing, *4OR: A Quarterly Journal of Operations Research*, 8 (4), 429–432.
- Mannino, C. , Mascis, A. (2009). Optimal real-time traffic control in metro stations, *Operations Research*, 57 (4), 1026–1039.

- D'Ariano, A., Pranzo, M., (2009). An advanced real-time train dispatching system for minimizing the propagation of delays in a dispatching area under severe disturbances, *Networks and Spatial Economics*, 9 (1), 63–84.
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