Algorithms based on time-expanded formulations for Train Timetabling Problems

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Outline

- Introduction to Train Timetabling
- Models based on time-expanded graphs
- Solution methods
- Generalization to skip-stop planning strategies and passenger-centric objectives

Train Timetabling

Railway Optimization Stages

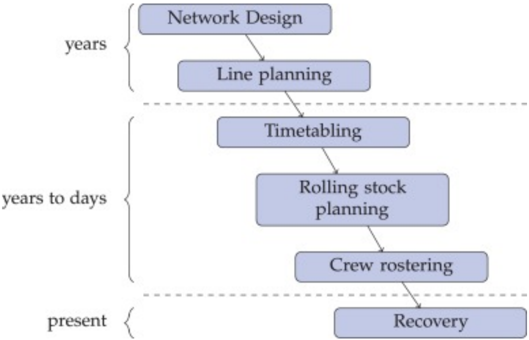


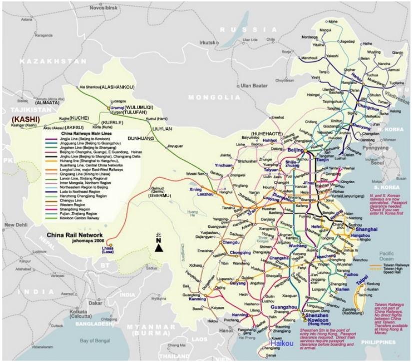
Figure from Lusby, R. M., Larsen, J., Bull, S. (2017).
A survey on robustness in railway planning. European Journal of Operational Research.

Train Timetabling

- It consists of finding an optimal schedule of trains in a railway network satisfying:
  - safety regulations (e.g., minimum headway times between consecutive trains on the same track) and
  - operational constraints (e.g., running times, dwell times, station capacity)
- The schedule is defined by the departure and arrival times of trains at all visited stations
- The objective function depends on the railway company (e.g., schedule as many trains as possible)

Railway infrastructure

- The railway infrastructure consists of a network with:
  - nodes: represent the locations where the trains may interact
  - tracks: connect the nodes and are used by the trains to travel from one node to the next one



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  1. A set of train lines (a route between an origin and a destination station with a specific stopping pattern) and a frequency of the train line
  2. An ideal timetable for each train provided by the Train Operator that specifies the departure and arrival times at each visited station of the railway network

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  - First the non-periodic problem (scheduling trains for a day) and then a periodic problem (scheduling trains for one hour)

Models based on time-expanded graphs

Non-periodic Train Timetabling - one-way line

- $S = \{1, \ldots, s\}$: set of stations
- T: set of trains each with:
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- The goal is to maximize the total importance of the scheduled trains and minimize the changes to the ideal timetables

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- Lower and Upper limits are imposed for these changes:
  - maximum shift at the departure station for each train
  - maximum total stretch

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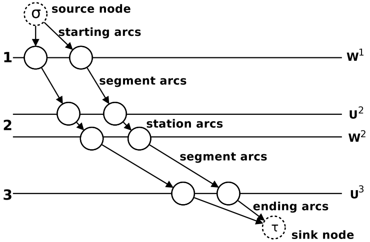

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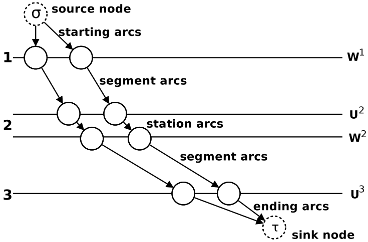

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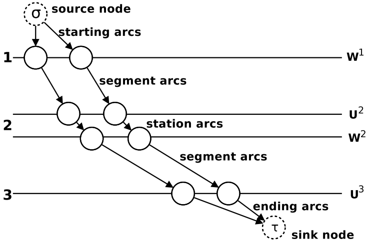

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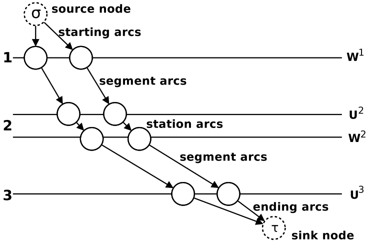

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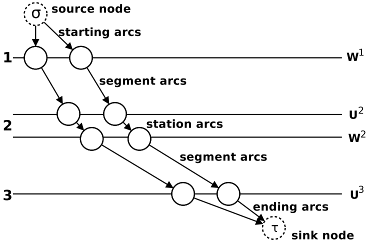

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A path in G from σ to τ corresponds to a timetable for a train

An example

Stations	Ideal Timetable A		Ideal Timetable B		Ideal Timetable C	
	Arr. Time	Dep. Time	Arr. Time	Dep. Time	Arr. Time	Dep. Time
1		9:00		9:00		
2	9:05	9:07	9:10	9:12		
3	9:18		9:30	9:35		9:33
4			10:00	10:03	10:02	10:07
5			10:20		10:24	

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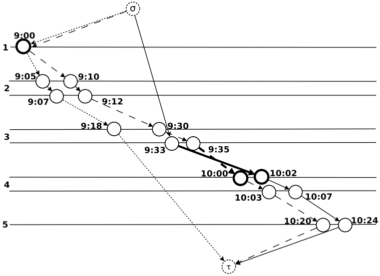

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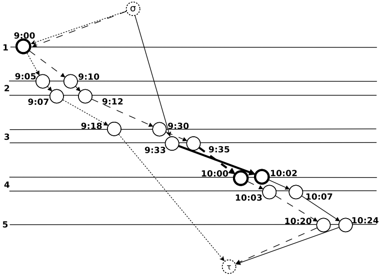

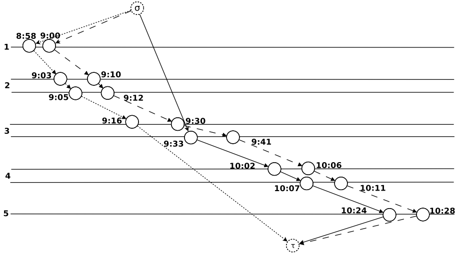

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$$\max \sum_{t \in T} \sum_{a \in A^t} p_a x_a$$

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$$\sum_{a \in \delta_t^-(v)} x_a = \sum_{a \in \delta_t^+(v)} x_a, \qquad t \in T, v \in V \setminus \{\sigma, \tau\},$$

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$$x_a \in \{0, 1\}, \qquad a \in A.$$

- $\mathcal{C}$: family of maximal subsets C of pairwise incompatible arcs

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- $\mathcal{IP}$: family of maximal subsets $\mathcal{I}$ of pairwise incompatible paths with incompatibility expressed separately for each station

Solution Methods

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  - Local search procedures to improve the solution found

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- Constraint separation is applied

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- Branching is applied on the choice of the arcs in the graph
- Constructive heuristics: LP-based fixing of paths or arcs in the graph

Generalization to include additional real-life features

Skip-stop planning strategies

Skip-stop planning strategies[1]

[1] F. Jiang, V. Cacchiani, P. Toth. Train Timetabling by Skip-Stop Planning in Highly Congested Lines. Transportation Research Part B, 104, 149-174, 2017.

Skip-stop planning strategies[1]

- An additional change to the ideal timetables: it is possible to skip a stop (= not schedule a stop)

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- Two sets of trains:
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- Acceleration and deceleration times must be taken into account
- maximum number of stops that can be cancelled per train
- no shift for the existing trains

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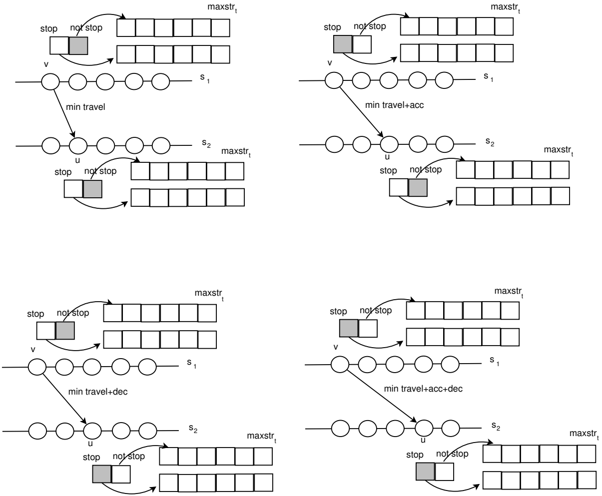
Solution method

- ILP arc-model with additional constraints
- Lagrangian-based heuristic algorithm
- Skip-stop strategies (with acceleration and deceleration) are handled by the Dynamic Programming algorithm

Dynamic Programming algorithm



Computational experiments - case study

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- the maximum number of stops that can be cancelled per train is set to 1
- the maximum stretch is set according to the origin-destination of the train
- the maximum shift is set to ± 10, ± 20 or ± 30 minutes

Computational experiments adding new trains

#trains	shift	#sched	travel	stretch	profit	gap%	time (s)
346 sh±10	109	328(0)	45829	1132(737)	986571	3.72	3857
346 sh±20	294	333(0)	45827	1142(740)	996286	2.98	6153
346 sh±30	415	336(1)	45681	1161(689)	998975	2.95	9732

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346 sh±30	415	337(0)	45752	1192(664)	1003265	2.55	2	11112

Table: With stop skipping

Passenger-centric objectives

Passenger-centric objectives[2]

- Line Planning Problem → frequency of trains for each line in the network

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- Passengers transfer between trains of different lines to reach their destination
- Therefore, trains of different lines have to be synchronized effectively

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- Synchronize trains and achieve regularity $\rightarrow$ minimize the total perceived passenger travel time: in-train time + transfer time + transfer penalty + adaption time (waiting time at the origin station)

Passenger-centric timetabling

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$$\text{Min}_\pi \sum_{k \in \mathcal{OD}} d_k \cdot R_k(\pi)$$

Such that π is a feasible timetable

passengers take best routes with respect to π

$R_k(\pi)$ avg. perceived travel time of one passenger of OD-pair k $\quad \forall k \in \mathcal{OD}$

Average perceived travel time

$$R_k(\pi) = \frac{1}{d_k} \sum_{v \in V^k} d_k \cdot \frac{L_v^k}{H} \cdot (\gamma_w \cdot W_v^k + Y_v^k) = \frac{1}{H} \sum_{v \in V^k} L_v^k \cdot (\gamma_w \cdot W_v^k + Y_v^k)$$

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- L_v^k: time interval between event v and the previous departure event of a route for OD-pair k
- The total number of passengers of OD-pair k arriving in each interval L_v^k is $d_k \cdot \frac{L_v^k}{H}$

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  - Evaluate the impact on passenger perceived travel time → feedback mechanism

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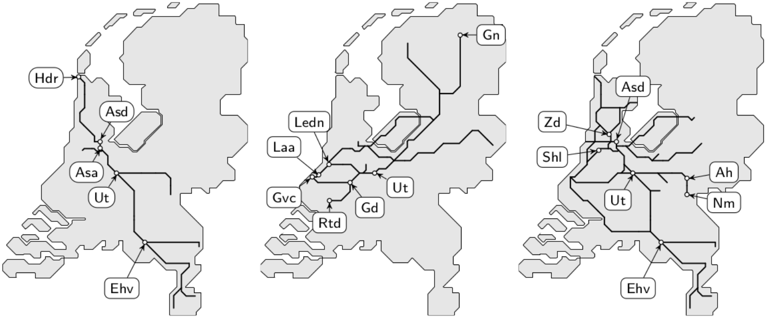
- We identify the OD-pairs that got the largest worsening
- We modify the profit structure by penalizing more the shift at origin and intermediate stations where the service was not regular
- Apply again the Lagrangian Heuristic

Computational experiments - case study

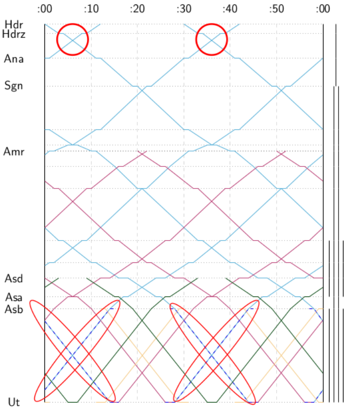
Three case studies of the Dutch railway network (lines of 2019) and one hour period:

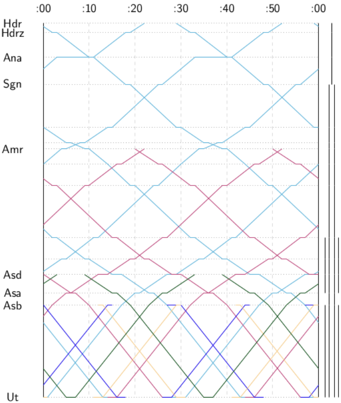
- A2: 34 stations, 20 trains, 891 OD-pairs.
- Rotterdam-Groningen: 77 stations, 60 trains, 3810 OD-pairs.
- Extended A2: 140 stations, 88 trains, 11121 OD-pairs.



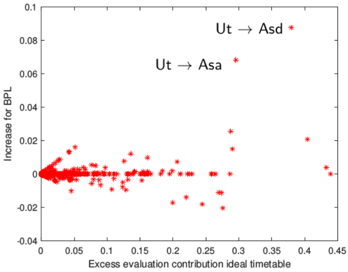
(a) A2 (b) Rotterdam-Groningen (c) Extended A2

A2 instance: ideal vs feasible timetable





A2 instance: before and after feedback



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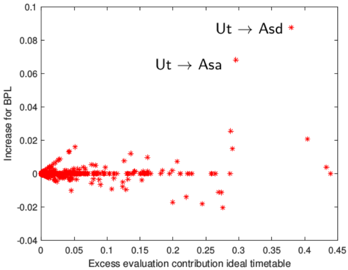

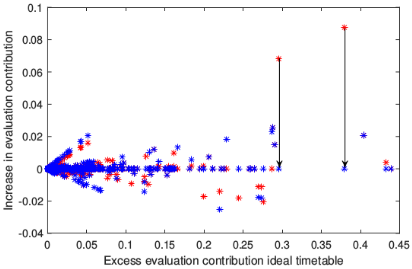

Comparison

Instance	Approach	Evaluation value	Time (hours)
A2	Ideal + LH	100.18	$2 + 0.03$

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	Ideal + LH	100.18	$2 + 0.03$
	Ideal + LH + FB	100.10	$2 + 0.11$
A2			

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A2	Ideal + LH	100.18	2 + 0.03
	Ideal + LH + FB	100.10	2 + 0.11
	Full PESP		
	- After 2.11 hours	105.80	2.11

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	- After 8 hours	104.88	8
	Lower bound CPLEX	97.09	
Rotterdam Groningen	Ideal + LH	100.59	$4 + 0.06$
	Ideal + LH + FB	100.55	$4 + 0.18$
	Full PESP		
	- After 4.18 hours	105.64	4.18
	- After 16 hours	103.69	16
	Lower bound CPLEX	92.72	

Comparison

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A2	Ideal + LH	100.18	2 + 0.03
	Ideal + LH + FB	100.10	2 + 0.11
	Full PESP		
	- After 2.11 hours	105.80	2.11
	- After 8 hours	104.88	8
	Lower bound CPLEX	97.09	
Rotterdam Groningen	Ideal + LH	100.59	4 + 0.06
	Ideal + LH + FB	100.55	4 + 0.18
	Full PESP		
	- After 4.18 hours	105.64	4.18
	- After 16 hours	103.69	16
	Lower bound CPLEX	92.72	
Extended A2	Ideal + LH	101.51	4 + 0.14
	Ideal + LH + FB	101.28	4 + 0.49
	Full PESP		
	- After 4.49 hours	-	4.49
	- After 16 hours	-	16
	Lower bound CPLEX	93.00	

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- $\rightarrow$ Andrea D'Ariano will talk about efficient methods for train rescheduling during rail operations

Thank you for your attention